TABLE 2.—*Computations of the atmospheric turbidity factor, β, and the water-vapor content of the atmosphere, w, from screened solar radiation measurements*

[From measurements at Washington, D.C. April 18, 1932]

Air mass m	Atmospheric turbidity β								Water-vapor content of the atmosphere, w (cm)					
	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)[b]	(15)[c]
	I_m	$\frac{I_r}{0.878}$	$\frac{I_y}{0.889}$	(2−3)[a]	(4−3)[a]	β from (5)	β from (6)	Mean (7)+(8)	I_m (w=0)	I_m	(10)−(11)	w (Cm)	Mm	Mm
									Percentage of solar constant					
3.99	0.881	0.700	0.790	0.183	0.091	0.077	0.098	0.088	0.490	0.459	0.031	0.16	0.70	----------
3.23	1.002	.737	.870	.268	.133	.065	.072	.078	.592	.522	.070	.39	1.75	----------
3.10	1.027	.747	.881	.283	.135	.062	.074	.068	.602	.536	.066	.35	1.57	----------
2.47	1.118	.778	.939	.343	.161	.060	.064	.062	.668	.582	.086	.49	2.20	----------
2.37	1.137	.782	.943	.359	.161	.058	.071	.064	.672	.592	.081	.47	2.11	3.63
1.83	1.223	.834	1.016	.393	.182	.064	.061	.068	.719	.637	.082	.50	2.24	----------
1.79	1.229	.837	1.022	.396	.185	.072	.069	.070	.719	.640	.079	.49	2.20	----------
1.57	1.296	.856	1.048	.444	.193	.063	.077	.065	.753	.675	.078	.49	2.20	----------
1.52	1.303	.859	1.060	.448	.201	.065	.060	.066	.757	.679	.078	.50	2.24	----------
1.44	1.326	.862	1.065	.469	.203	.059	.060	.060	.785	.691	.094	1.00	4.48	----------
1.40	1.338	.863	1.069	.480	.206	.053	.061	.057	.785	.697	.088	.95	4.26	----------
1.32	1.354	.843	1.073	.516	.230	.043	.025	.034	.883	.705	.128	1.41	6.32	----------
1.15	1.428	.913	1.124	.520	.211	.056	.073	.064	.805	.744	.061	.69	3.09	3.15
1.46	1.284	.846	1.053	.442	.207	.074	.055	.064	.765	.669	.096	1.01	4.49	----------

[a] Corrected for mean solar distance.

[b] Surface water-vapor pressure $= \frac{w}{.223}$

[c] From psychrometer measurements.

CONSERVATION OF ANGULAR MOMENTUM, OR AREAS, AS APPLIED TO AN AIRPLANE EN ROUTE TO THE POLE

By W. J. HUMPHREYS

[Weather Bureau, Washington, April 1933]

When a freely moving object is held on its course by a pull or push continuously directed to the same point, as illustrated by a planet tracing its orbit about the sun under the force of gravity, the areas swept over by the straight line, or radius vector, connecting the center of attraction with the moving object during different equal intervals of time are equal to each other, however near to or far from that center the object may be. This is the law of the conservation of areas, or conservation of angular momentum. The same law applies to the atmosphere, barring the effects of friction and turbulence, when forced to change latitude. In this case the radius vector is the perpendicular from the place occupied onto the axis of the earth, or radius of the small latitude circle through the place in question.

Rigid proofs of these laws are well known, though few books contain them in detail. They are based on, or in keeping with, the conservation of energy, hence without exception and not in the least contravened by the fact that the air in high latitudes often is just as quiet as that of any other part of the world. However, one may accept the logical proofs of all these statements and still be puzzled by the fact that one can fly to either pole of the earth, as has been done, without being driven into a dizzy west-to-east spin about it.

If the law of the conservation of areas is true, and if the force driving the plane seems all the time directed strictly towards the pole, then why is it that the plane, instead of spinning around the world from west to east, at an ever-increasing speed, keeps to the same meridian?

The law, as stated, is true, and the plane is kept from speeding eastward by a counter force—the driving force is not strictly towards the pole.

How great then is this counter force?

Let the conditions be:

Latitude of plane, $\lambda = 80°$.

West-to-east velocity of plane same as earth beneath, $u = \frac{2\pi R \cos\lambda}{T} = \frac{2\pi r}{T}$.

R = Radius of the earth.

T = Time of rotation of the earth (siderial day) = 86,164 seconds.

r = Radius of circle of latitude at latitude λ.

Velocity of plane towards adjacent pole, $v = 200$ kilometers per hour.

By the law of the conservation of areas, ur = constant.

Hence $\frac{du}{u} = -\frac{dr}{r}$

Then, if s is a distance along a meridian (approach to pole positive) the west-to-east acceleration

$$\frac{du}{dt} = -\frac{u}{r}\frac{dr}{dt} = \frac{u}{r}\frac{ds}{dt}\sin\lambda = \frac{u}{r}v\sin\lambda = \frac{u}{R}v\tan\lambda$$

When u is equal to the west-to-east velocity of the surface of the earth at the place in question, that is, when the plane has no motion across the meridian, the last expression,

$$\frac{u}{R}v\tan\lambda = \frac{2\pi v\sin\lambda}{86164}$$

On substituting the value of v in terms of centimeters and seconds, and the value of $\sin\lambda$ for $\lambda = 80°$, it appears that, under the conditions stated, $\frac{du}{dt} = 0.4$ cm/sec.^{2}, nearly, or about 1/2450 part of gravity acceleration. The maximum value, as the pole is reached, is but little greater.

Now the ratio of thrust to the lift, in the case of an airplane, is, roughly, 1 to 8. Hence, in the above case, an east-to-west push equal to about one 300th that of the poleward thrust would fully counteract the effect of the law of the conservation of areas and keep the plane on the same meridian. This would be accomplished by heading the plane rather less than one fifth of a degree west of the true meridian, an amount that would seem to the aviator, if noticed at all, as a mere drift correction.

The law of the conservation of areas is true, nevertheless it does not perceptibly interfere with the interzonal travel of airplanes, even to the poles of the earth.